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Plane Electromagnetic Wave Interaction with Nonuniform Bragg gratings: non-traditional approach

Introduction

The problem of plane electromagnetic wave interaction with modulated media is topical in all range of electromagnetic wave frequencies. Modulation of medium means that permittivity or absorption, or both of these are variable parameters of medium along the fixed direction. In the case of periodic modulation the well-known analytical results have been obtained for weak modulation [1, 2]. But nowadays the investigation of periodical and nonuniform modulated media is of great scientific and practical interest. Nonuniformity here means that geometry and physical properties of a medium are varied along the axis of wave propagation. Fiber Bragg gratings belong to the wide class of such type modulated media. They are devices with many identified and potential areas of application. They can serve as multichannel add/drop filters, spectrum flattening filters, stretchers and compressors, dispersion compensators, highly sensitive sensors, etc. For many of these applications, the gratings are nonuniform Bragg gratings.

For theoretical investigation of uniform Bragg gratings several methods have been developed. One of the widely used methods is the well-known coupled-wave theory [3, 4]. For periodical structures transfer matrix method [5] and Bloch wave analysis [1, 6, 7] have been used successfully.

Other less employed methods have been devoted and developed for uniform Bragg gratings analysis. They are the Rourd's method [8], a discrete-time approach based on a digital signal processing formulation [9] and a Hamiltonian formulation for coupled-wave equations [10].

For nonuniform Bragg gratings transmittive and reflective properties investigation the extension of the methods described above has been used. They are the extensions of transfer matrix method [5], Bloch wave analysis generalization [11], coupled-wave theory [12] and WKB [13] methods extension. A variational technique [14] and a Hamiltonian approach for nonuniform Bragg grating [15] analysis have been employed also.

The all generalized methods for nonuniform Bragg gratings investigation have a restriction on the amplitude of modulation. The reason is that for these methods it's significant that the wave equation solution is searched as a superposition of counter-propagating waves. The last condition badly complicates the problem solving in the case of nonperiodically and strongly modulated media.

To avoid this drawback in the present paper a new non-traditional approach is presented to investigate plane electromagnetic wave interaction with nonuniform Bragg gratings. This approach is based on the fact, that a solution of the wave equation in a modulated media is searched in the form of a single expression, but not in the form of generally accepted counter-propagating waves. In this case the correct boundary problem is solved not only for linear but for nonlinear media as well. Furthermore, this approach permits to carry out the investigation for media with losses (or gain) and doesn't need any preliminary assumptions concerning the form of

wave equation solution (i.e. the form of travelling waves, counter-propagating waves, exponentially increasing or decreasing waves or others in the case of nonlinearity).

Such a representation of wave equation solution is very effective to investigate electromagnetic wave interaction with arbitrary modulated media (including nonlinear) and permits to solve numerically diverse problems easily and comparatively rapidly by using of a well-known Runge-Kutta method of numerical integration.

Principal Expressions

In the present work the normal incidence of plane electromagnetic wave on an apodized Bragg grating is considered (see Fig.1).

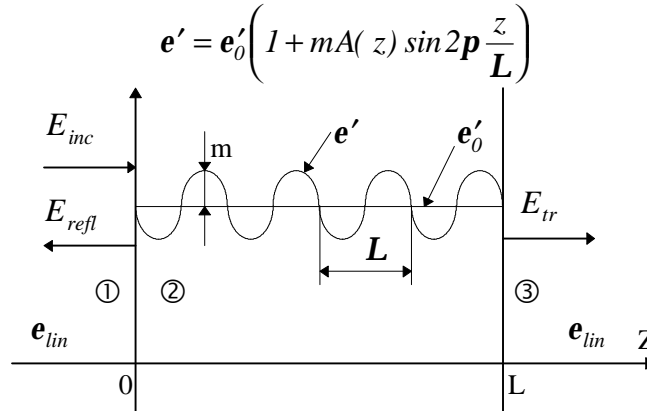


Fig.1. Bragg grating one-dimensional configuration

Here, L is the length of medium, ϵ_{lin} and ϵ' are the permittivities out and within the modulated medium, respectively; λ_0 and Λ are electromagnetic wave and the permittivity distribution wavelengths, respectively; m is the amplitude of modulation; $A(z)$ describes the changing of modulation depth along the structure (i.e. apodization).

The linear polarization of the incident electric field is assumed.

As it is known, from Maxwell's equations in the case of isotropic medium for one of the field components E_x the wave equation can be obtained:

$$\frac{\nabla^2 E_x(z, t)}{\nabla z^2} + \mu_0 \epsilon_a \frac{\nabla^2 E_x(z, t)}{\nabla t^2} = 0, \quad (1)$$

where $\mu_0 = 4\pi \cdot 10^{-7}$ (H/m) is the magnetic permeability of the free space, ϵ_a - is the permittivity of medium. This wave equation is valid also for inhomogeneous media, when the condition $\vec{E} \perp \text{grad} \epsilon_a$ is satisfied.

The solution of a wave equation (1) can be searched in the form of a single expression without dividing on counter-propagating parts:

$$E_x(z, t) = U(z) \cos(\omega t - S(z)), \quad (2)$$

where $U(z)$ and $S(z)$ - are the real quantities called amplitude and phase functions, respectively, and describe the electromagnetic wave amplitude and phase without the requirement of the superposition principle performing.

Such a form of solution (2) of wave equation (1) has been proposed by a number of authors [16-18] and further has been extended, basically, to investigate different types of multilayer structures [19-22].

The solution (2) can be represented as:

$$E_x(z, t) = \frac{\dot{E}_x(z)}{2} e^{j\omega t} + \frac{\dot{E}_x^*(z)}{2} e^{-j\omega t}, \quad (3)$$

where

$$\dot{E}_x(z) = U(z) e^{-jS(z)} \quad (4)$$

and the sign * shows a complex conjugation.

Substituting expression (3) into the wave equation (1) gives the following Helmholtz's equation for $\dot{E}_x(z)$:

$$\frac{d^2 \dot{E}_x(z)}{dz^2} + \omega^2 \mathbf{m}_0 \tilde{\epsilon}_a \dot{E}_x(z) = 0 \quad (5)$$

Here $\tilde{\epsilon}_a = \mathbf{e}'_a + j\mathbf{e}''_a$ is the complex permittivity.

The time dependence $e^{j\omega t}$ will be implied but suppressed throughout the analysis.

On substituting expression (4) into equation (5) and after separation of real and imaginary parts the following set of equations is obtained:

$$\begin{cases} \frac{d^2 U(z)}{d(k_0 z)^2} - \frac{P^2(z)}{U^2(z)} + \mathbf{e}' \cdot U(z) = 0 \\ \frac{dP(z)}{d(k_0 z)} = \mathbf{e}'' \cdot U^2(z) \end{cases} \quad (6)$$

where $k_0 = \omega \sqrt{\mathbf{e}_0 \mathbf{m}_0}$ - is the free space propagation constant, $\mathbf{e}' = \mathbf{e}'_a / \mathbf{e}_0$, $\mathbf{e}'' = \mathbf{e}''_a / \mathbf{e}_0$, $\mathbf{e}_0 = 10^{-9} / 36\pi$ (F / m) and

$$P(z) = U^2(z) \frac{dS(z)}{d(k_0 z)} \quad (7)$$

is a quantity proportional to the power flux density along the Z-axis [17].

It is convenient to reduce the second-order system of differential equations (6) into a set of first-order equations, using the new variable $Y(z)$ given by the field derivative in the medium:

$$\begin{cases} \frac{dU(z)}{d(k_0 z)} = Y(z) \\ \frac{dY(z)}{d(k_0 z)} = \frac{P^2(z)}{U^3(z)} - \mathbf{e}'_0 \left(1 + mA(z) \sin \frac{2\mathbf{p}z}{L} \right) U(z) \\ \frac{dP(z)}{d(k_0 z)} = \mathbf{e}''_0 U^2(z), \end{cases} \quad (8)$$

This set of equations describes the spatial behaviour of electric field amplitude $U(z)$, its derivative $Y(z)$ and the power flux density $P(z)$ in the medium.

In describing of waves outside of the investigated structure (in linear media) the traditional counter-propagating waves approach is used. Inside the structure the form of solution given by equation (2) is used. The following relations may be written:

A). In the left half-space ($z < 0$) incident and reflected plane electromagnetic waves are expressed as follows:

$$\begin{aligned} E_{xinc} &= E_{inc} e^{-jk_0 \sqrt{\epsilon_{lin}} z} \\ E_{xref} &= E_{ref} e^{+jk_0 \sqrt{\epsilon_{lin}} z} \end{aligned}$$

where ϵ_{lin} is the permittivity of the outside linear medium.

On the illuminated surface at $z=0$ from the conditions of the tangential components of electric and magnetic fields continuity the reflection coefficient r

$$r = \frac{E_{ref}}{E_{inc}} = \frac{U^2(0) \cdot \sqrt{\epsilon_{lin}} - P(0) - jU(0) \cdot Y(0)}{U^2(0) \cdot \sqrt{\epsilon_{lin}} + P(0) + jU(0) \cdot Y(0)} \quad (9)$$

is obtained as a function of the field parameters at $z = 0$ [23].

B). In the right half-space ($z > L$) the transmitted field is expressed as follows:

$$E_{xtrans} = E_{tr} e^{-jk_0 \sqrt{\epsilon_{lin}} (z-L)} .$$

The ordinary boundary conditions of the continuity of the electric and magnetic field tangential components continuity at $z = L$ in this case can be written as follows:

$$U(z=L) = E_{tr} , S(z=L) = 0 , Y(z=L) = 0 , P(z=L) = U^2(L) \cdot \sqrt{\epsilon_{lin}}$$

Obviously, starting to solve the boundary problem from the nonilluminated side of the structure ($z = L$), we reduce it to a Cauchy problem, i.e. to an initial value problem.

Setting up the initial values for U, Y and P at $z = L$, the numerical integration of the differential equations set (8) is carried out towards the illuminated border ($z = 0$). Integration process carried out towards to the illuminated side of the considered structure permits to restore at the end of calculation the reflection coefficient r from (9) by using of the obtained $U(0), Y(0), P(0)$ values.

Numerical Results

As an example, we have computed the Bragg grating's reflectivity ($|r|^2$) in the case of raised-square-cosine -apodization with zero-dc index change. $A(z) = \sin^2(\mathbf{p}z/L)$ (see Fig.2).

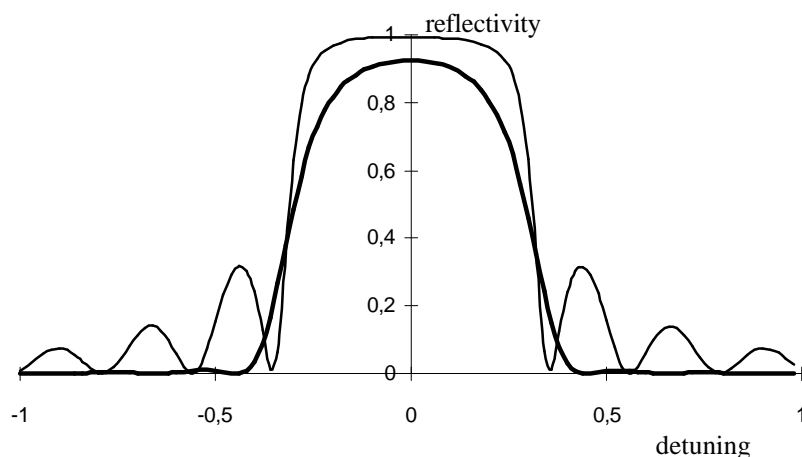


Fig. 2. Reflectivity of square cosine apodized (thick line) and uniform (thin line) Bragg gratings versus the detuning. $m = 0.005$, $L/L = 800$.

On horizontal axis the detuning $x = \frac{1}{m} \left(1 - 2L\sqrt{\epsilon'_0} / l_0 \right)$ is placed.. As it is seen from the Fig.

2 square -cosine- apodization provides the essential depressing of sidelobes. The sidelobes elimination is very important for gratings application as narrow-band filters in fiber-optics communication systems.

The other obtained results for different types of nonuniform Bragg gratings are in good agreement with the results obtained by using of traditional methods of investigation [24, 25]. From this point of view we can assert that the non-traditional approach presented in this paper is capable to compute an arbitrary strong Bragg gratings. This assurance comes from the fact that the presented approach has no restrictions on the amplitude and character of modulation, and inherently is exact. It is important also to notice that this approach allows to take into account losses, gain [26] and Kerr-type nonlinearity as well.

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